GROUP THEORY in Ankara

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2-3 May 2019

in honour of
Mahmut Kuzucuoğlu's
60th birthday

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

TÜRK MATEMATİK DERNEĞİ

Group Theory in Ankara
2 - 3 May 2019
Middle East Technical University

Welcome to the conference "Group Theory in Ankara, 2019" in honor of Prof. Mahmut Kuzucuoğlu's 60th birthday. Thank you for joining us for a celebration of the mathematical career of Prof. Kuzucuoğlu.

We are delighted to see a high number of participants. This is not only a sign of the significant interest in the theory of groups but also a sign of gratitude for Prof. Kuzucuoğlu. Each of us has a reason to attend this conference. Some of us are Kuzucuoğlu's students; some of us his friends. In either case, we appreciate the scientific contributions of Prof. Kuzucuoğlu as well as his friendship.

As we started organizing this conference several months ago, we aimed to bring together a community of researchers at all stages focusing on group theory. We are grateful to all the participants and speakers of this conference. Special thanks to the ones who come from long distances; USA, Russia, Poland, Italy, and Iran. We appreciate their support for the conference.

We want to thank Middle East Technical University and Turkish Mathematical Society for supporting this conference.

We hope you have a good time at the conference.

2 May 2019
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Some steps towards unraveling the non-associativity enigma in algebraic structures

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Abstract

Non-commuatative algebraic structures are certainly more complicated and less known than commutative ones. In recent years, to overcome this difficulty, some of the researchers in these areas have invented and applied some new tools constructed with combinatorial, computational, probabilistic and graph theoristic methods. For example, to characterizing some classes of groups, such as finite non-abelian simple groups, they have defined and used some associated graphs to finite groups like prime graph and non-commuting graph. As well as, commutativity degree of a finite group is a probabilistic tool for determining and specifying some classes of groups such as nilpotent, supersolvable and solvable finite groups. Then some of these tools have been generalized to other algebraic structures such as rings and semigroups.

These methods have been useful, easy to handle and more transparent and explicit than abstract ones, and in these early years of 21^{st} century have contributed much more to new developments and advances in various branches of algebras.

Analogously, non-associative algebraic structures are much more complicated and less known than associative ones. But, there is almost no attempt to create and use combinatorial, probabilistic and graph theoristic methods to study and specify them better and more complete.

In this talk, we are going to introduce some probabilistic, combinatorial and graph theoristic methods to study some non-associative algebraic structures, which can enable us to characterize and determine various classes of them and to better understand their interior structures.

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Prof. Mahmut Kuzucuoğlu and his contributions to group theory

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Prof. Dr. Mahmut Kuzucuoğlu was born on 15th November, 1958, in Denizli, an Aegean province of Turkey. He graduated from Denizli High School in 1976 and started studying mathematics in Middle East Technical University. He obtained his BS, and MSc degrees in METU, in 1981 and 1984 respectively. After getting another Master's degree from University of Toledo in 1985, he got his PhD from the Victoria University of Manchester in 1987. His doctoral supervisor was Prof. Brian Hartley, who was one of the leading group theorists of the late 20th century. Together with B. Hartley he proved that in an infinite simple locally finite group, the centralizer of any element is infinite (see [2]). Moreover, by using this result he proved that there exists no simple locally finite minimal non-FC groups, together with R. Phillips (see [1]).

After his PhD, he returned Turkey and started working at Middle East Technical University as an assistant professor. He became an associate professor in 1993 and a full professor in 1999. He has visited many research institutions and universities including Krasnoyarsk State Academy, Oberwolfach Mathematics Institue, Freiburg Institute of Advanced Studies, University of Napoli and University of Freiburg.

His research interests are usually related to infinite group theory, and he is one of the experts of locally finite groups. He has proved numerous of results about centralizers of elements in simple locally finite group, minimal non-FC groups, barely transitive groups, existentially closed groups and direct limits of infinite symmetric groups. He has published more that 35 research articles in various prestigious scientific journals, collaborated with many mathematicians and until 2019 he supervised 4 doctoral students.

In this talk we will talk about the important contributions of Prof. Dr. Kuzucuoğlu to group theory and we will present some of his very famous results. We add a list of his publications.

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Embedding properties in uncountable groups

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The aim of this talk is to show that in uncountable groups, as well as in groups of infinite rank, the behaviour of small subgroups is often neglectable.

Frobenius groups of automorphisms with almost fixed point free kernel

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Let FH be a Frobenius group with kernel F and complement H, acting coprimely on the finite solvable group G by automorphisms. We prove that if $C_G(H)$ is of Fitting length n then the index of the n-th Fitting subgroup $F_n(G)$ in G is bounded in terms of $|C_G(F)|$ and $|F|$. This generalizes a result of Khukhro and Makarenko which handles the case $n = 1$.

[*] Joint work with Gülin Ercan

Properties of groups and Lie algebras of infinite matrices

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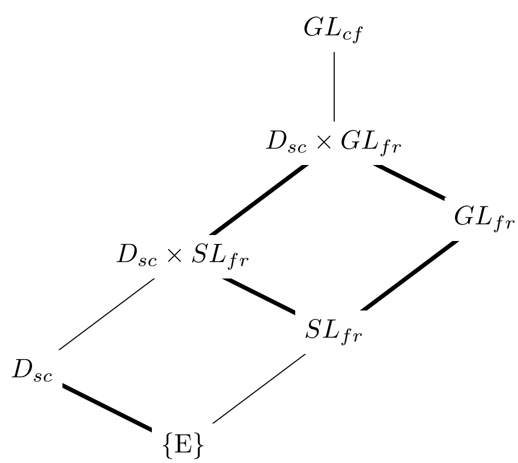
Description of normal subgroups is a fundamental problem in group theory. The classical result due to Jordan, Burnside, Dickson, says that every normal subgroup of $GL(n, K)$ (K - a field, $n \geq 3$) which is not contained in the center, contains $SL(n, K)$. We extend this result, giving description of normal subgroups in the group $GL_{cf} = GL_{cf}(\mathbb{N}, K)$ of invertible column-finite infinite matrices over K indexed by $\mathbb{N}$ (see [4]).

The lattice of normal subgroups of GL_{cf} "modulo the center" is shown in the figure below. The thin line between subgroups H_1 and H_2 ($H_1 \leq H_2$) means that the factor group H_2/H_1 is simple, the thick line means that the factor group H_2/H_1 is isomorphic to K^*. D_{sc} denotes the subgroup of scalar matrices, GL_{fr} the subgroup of all matrices which differ from identity matrix in only finite number of rows, SL_{fr} the subgroup of GL_{fr} with all matrices having in left upper corner the matrix with determinant 1.



Alex Rosenberg in [5] gave description of normal subgroups of $GL(V)$, where V is a vector space of any infinite cardinality dimension. In countable case his result is incomplete. He proved only that $D_{sc} \times GL_{fr}$ is maximal normal subgroup of GL_{cf} and thus $GL_{cf}/(D_{sc} \times GL_{fr})$ is simple.

Our results fill this gap giving the full description of the lattice of normal subgroups of the group of infinite column-finite matrices indexed by positive integers over any field. In the proof we use some facts from [1], [2]. We note

* Joint work with M. Maciaszczyk and S. Żurek.

that similar description of ideals of the Lie algebra of infinite column-finite matrices over any field was obtained in [3].

In our talk we give a survey of these results.

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Problems on structure of finite quasifields and projective planes

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Geometric properties of projective planes are studied together with algebraic properties of their coordinatizating sets [1]. It is well known that a finite Desarguesian projective plane is coordinatized by a field, and weakening of commutativity and associativity for multiplication leads to translation planes which are coordinatized by quasifields. A quasifield with two-sided distributivity is considered to be a semifield, it coordinatizes a semifield plane. A detailed review on semifields, quasifields and correspondent projective planes is presented in [2]; however, many problems are little studied.

The solvability problem for full collineation group of a projective plane which is coordinatized by a finite proper semifield ([1], see also question 11.76, 1990, [3]) is unsolved still.

Wene's hypothesis (1991) on right- or left-primitivity of a finite semifield was refuted by I. Rúa [4] who provided the couter-examples of order 32 (2004) and 64 (2007). Nevertheless, all known examples of semifields of small order (up to 243) have a left-cyclic base and 1-generated loop of non-zero elements.

The following problems for finite proper quasifields were presented in 2013 by V.M. Levchuk at algebraic research seminar in Moscow State University and also were formulated in [6].

(A) Enumerate maximal subfields and their orders.

(B) Find a finite quasifield S with non-1-generated loop S^*. The hypothesis is as follows: a loop S^* of any finite semifield S is 1-generated.

(C) What spectra of a loop S^* are possible, if S is a finite quasifield or a semifield?

(D) Find the automorphism group $\mathrm{Aut}\, S$.

We have found [5, 6, 7] the solution of problems A–D for all semifields of order 16, as well as for for counter-examples Knuth–Rúa semifield of order 32, Hentzel–Rúa semifield of order 64, and some isotopism classes representatives for semifields and quasifields of small order.

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* Joint work with V. M. Levchuk

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Niltriangular subalgebra of Chevalley algebra and the enveloping algebras

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According to [1], an algebra $R = (R, +, \cdot)$ is called *an enveloping algebra* of Lie algebra L if the algebra $R^{(-)} := (R, +, \ *)$ for $a * b := ab - ba$ is isomorphic to L. (See also Lie-admissible algebras [2].) We study the case of Chevalley algebra L over a field K having a basis $\{e_r \ (r \in \Phi),\ h_s := e_s * e_{-s} \ (s \in \Pi)\}$ for a root system Φ and its basis Π.

By Chevalley's basis theorem [3, 4.2.1], for any $r, s \in \Phi$ we have $h_s * h_r = 0, \quad h_s * e_r = 2(r, s)/(r, r)\ e_r$ and, also,

$$e_r * e_s = N_{rs} e_{r+s} = -e_s * e_r \ (r + s \in \Phi), \quad e_r * e_s = 0 \ (r + s \notin \Phi \cup \{0\}),$$

where $N_{rs} = \pm 1$ or $|r| = |s| < |r + s|$ and $N_{rs} = \pm 2$, or (for type G_2) $N_{rs} = \pm 2$ or ± 3. An enveloping algebra for L depends from choice of signs of structural constants N_{rs}. These signs can be chosen arbitrary only for extraspecial pairs $r, s \in \Phi^+$, [3, 4.2.2]. Thus, an enumeration of enveloping algebras for L is reduced to a similar enumeration for *niltriangular subalgebra* $N\Phi(K)$ with the basis $\{e_r \mid r \in \Phi^+\}$. By [1, Proposition 1] we have

Proposition. *A K-algebra with the basis $\{e_r \mid r \in \Phi^+\}$ is an enveloping algebra of $N\Phi(K)$ if $e_r e_s = 0$ at $r + s \notin \Phi$, and for $N_{rs} \geq 1$ we have $e_r e_s = e_{r+s}$ and $e_s e_r = (1 - N_{rs}) e_{r+s}$.*

For $r, s \in \Phi^+$ we set $r \leq s$ if $s - r$ is a linear combination of simple roots with nonnegative coefficients. Distinguish ideals $Q(r) := \sum_{r<s} Ke_s$ and $T(r) := \sum_{r \leq s} Ke_s$ in a Lie algebra $N\Phi(K)$. Roots r and s are called *incident* ones if $s \leq r$ or $r \leq s$ hold (i.e., $T(r) \subseteq T(s)$ or $T(s) \subseteq T(r)$). Any set $\mathcal{L}$ of pairwise non-incident roots in Φ^+ is said to be *a set of corners in* Φ^+. If $H \subseteq \sum_{r \in \mathcal{L}} T(r)$ and the inclusion fails under every substitution of $T(r)$ by $Q(r)$, then $\mathcal{L} = \mathcal{L}(H)$ is said to be *a set of corners in H*.

An ideal H of Lie ring $N\Phi(K)$ is said to be *standard* if $H \supseteq Q(\mathcal{L})$. Problem of enumeration of all standard ideals of Lie algebras $N\Phi(K)$ of classical types over finite field $K = GF(q)$ [5] is solved [1] in Egorychev – Levchuk – Khodunja's theorem.

The algebra $NT(n, K)$ of all niltriangular $n \times n$-matrices over K is isomorphic to well-known enveloping algebra R of type A_{n-1} in Proposition 1. Thus, by Dubish – Perlis's theorem [4, Theorem 8], R is a *standard enveloping algebra*, i.e., all its ideal are standard. The constructed enveloping algebras of other types are non-associative.

A standard enveloping algebra R of Lie algebra $N\Phi(K)$ exists for each type, besides Lie types D_n $(n \geq 4)$ and E_n $(n = 6, 7, 8)$. Further we establish uniqueness of standard enveloping algebra $R\Phi_n(K)$ of $N\Phi(K)$ for type B_n (analogously, for type A_n and C_n) such that $R\Phi_n(K) \subseteq R\Phi_{n+1}(K)$ for any n. An enveloping algebra R of type D_n is determines uniquely as a subalgebra in $R\Phi_n(K)$ of type B_n with basis from elements e_r for all long roots r. In these cases uniquely choice of signs of structural constants N_{rs} corresponds of the representation in [6] of Lie algebra $N\Phi(K)$ of classical Lie types.

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On functions in periodic groups defined by element orders

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Let G be a periodic group. The aim of this talk is to present some results that answer to the question of what information about G can be derived by looking at the orders of its elements. In particular, if G is a finite group, we will study the function on the element orders of G defined by

$$\psi(G) = \sum_{x \in G} o(x),$$

where $o(x)$ denotes the order of the element x.

In [1] H. Amiri, S.M. Jafarian Amiri and M. Isaacs proved that if G has order n and C_n denotes the cyclic group of order n, then $\psi(G) \leq \psi(C_n)$, and $\psi(G) = \psi(C_n)$ if and only if $G \simeq C_n$. Other results have been obtained by H. Amiri, S.M. Jafarian Amiri, M. Amiri, Y. Marefat, A. Iranmanesh, A. Tehranian, R. Shen, G. Chen and C. Wu.

We will report some new results concerning the function ψ, jointly obtained with Marcel Herzog and Mercede Maj in the papers [2], [3], [4], [**?**]. In particular we will present some better upper bounds for $\psi(G)$ when G is not cyclic, and some results on the structure of G assuming some bounds on $\psi(G)$.

Some other functions on the orders of the elements of a group G will be also investigated.

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* Joint work with Marcel Herzog and Mercede Maj

Small doubling problems in some classes of groups

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Let G denote an arbitrary group. If S is a subset of G, then we write

$$S^2 = \{xy \mid x, y \in S\}.$$

A well-known problem in additive number theory is to find the precise structure of S, if S is a finite subset of G, and

$$|S^2| \leq \alpha|S| + \beta,$$

with α (the doubling coefficient) and $|\beta|$ small. Problems of this kind are called *inverse problems of small doubling type*.

Inverse problems of small doubling type have been first studied in the group of the integers by G.A. Freeman (see [1]), and then in arbitrary abelian groups by many other authors. More recently, this kind of problems in non-abelian groups have also been studied. In a series of papers with G.A. Freiman, M. Herzog, P. Longobardi, Y.V. Stanchescu, A. Plagne and D.J.S. Robinson we studied inverse problems of small doubling type in an orderable group (see for example [2], [3], [4]). The aim of this talk is to discuss some problems and some new results related to this topic in some larger classes of torsion-free groups.

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Absolute centre, autocommutator and central autocommutator subgroups of a group

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The absolute centre $L(G)$ of a group G is the subgroup of all elements of G, which are fixed by every automorphism of G, and an automorphism of G is called autocentral if it acts trivially on the factor group $G/L(G)$.

We have introduced the notion of central autocommutator subgroup of a given group G and obtain some informations on the concept. Also some new results concerning the central kernel subgroup of G, which was first introduced by F. Haimo in 1955, are given.

In fact, the analogue of Schur's result is proved and construct some upper bounds for the order of central kernel and central autocommutator subgroups of G in terms of the order of central kernel quotient of G.

The conjugacy problem in groups with quadratic Dehn function.

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The first examples of finitely presented groups with decidable word problem and undecidable conjugacy problems were found by P.S. Novikov and W.W. Boone in 50'-s. Dehh function $d(n)$ can be regarded as a measure of the complexity of a finitely presented group, and the first examples of the groups with undecidable conjugacy problem have exponential Dehn functions. It is well known, that the conjugacy problem is decidable if $\liminf_{n\to\infty} d(n)/n^2 = 0$. With M.V. Sapir, we have constructed finitely presented groups with quadratic Dehn function and undecidable conjugacy problem. This answers E. Rips' question of 1994.

Cellular automata and surjunctive groups

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This talk is divided into three parts: in the first part we give some historical motivations of the theory of cellular automata emphasizing on Conway's *Game of Life* and the classical *Symbolic Dynamic*. In the second part, we give a new definition of a cellular automaton as a uniformly continuous map $A^G \to A$. Here, A is an arbitrary alphabet set and G is a group. The set A^G is considered with its pro-discreet uniform structure and the uniform structure on A is discrete. We show that the set of all such uniformly continuous maps has a natural monoid structure which is isomorphic to $CA(G, A)$, the classical monoid of cellular automata over group G with alphabet A. We use this result to re-prove the known theorem of Curtis-Hedlund:

Theorem. *Let A be finite and $T : A^G \to A^G$ be continuous and G-equivariant. Then T is a cellular automaton.*

In the final part, we focus on *surjunctive* group. A group G is called surjunctive, if for any finite set A, any injective cellular automaton $T : A^G \to A^G$ is also surjective. It is known that every locally finite group and every locally residually finite group is surjunctive. It is not known that if the direct product of two surjunctive groups is again surjunctive. We prove that

Theorem. *Any semi-direct product $L \ltimes S$ with L locally finite and S surjunctive, is surjunctive.*

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